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V Semester B.A./B.Sc. Examination, December - 2019
(CBCS) (F+R) 2016-17 and Onwards)

MATHEMATICS - V

Time : 3 Hours

Max. Marks : 70

Instruction : *Answer **all** questions.*

PART - A

Answer **any five** questions. **5x2=10**

1. (a) Give an example of
   (i) a ring with zero divisor
   (ii) a non-commutative ring with unity

   (b) In a ring (R, +, ·) prove that $a\cdot(b-c)=a\cdot b-a\cdot c \ \forall a, b, c \in R$.

   (c) Define principal and maximal ideals of a ring R.

   (d) Find the maximum directional derivative of $\phi=x^3y^2z$ at the point (1, −2, 3).

   (e) If $\vec{f}=3x^2\hat{i}+5xy^2\hat{j}+xyz^3\hat{k}$ then, find $\operatorname{div}\vec{f}$ at (1, 2, 3).

   (f) Evaluate : $\Delta^4(1-ax)(1-bx)(1-cx)(1-dx)$.

   (g) Write Lagrange's Interpolation formula for unequal intervals.

   (h) Using Trapezoidal rule, evaluate $\int_0^6 f(x)\,dx$ given :

x	0	1	2	3	4	5	6
$f(x)$	0.146	0.161	0.176	0.190	0.204	0.217	0.230

PART - B

Answer **two** full questions. **2x10=20**

2. (a) Prove that the set R={0, 1, 2, 3, 4, 5} is a commutative ring w.r.t. $\oplus_6$ and $\otimes_6$ as two compositions.

   (b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in it.

OR

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3. (a) Prove that the necessary and sufficient conditions for a non-empty subset S to be a subring of R, are :
(i) $S+(-S)=S$ (ii) $SS \subseteq S$

(b) Define the right and left ideals of a ring R. Show that the subset $S=\left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} | a, b \in z\right\}$ of $M_2(z)$ is a left ideal but not a right ideals of $M_2(z)$.

4. (a) (i) If 'a' is an element of a commutative ring R, then prove that $aR=\{ar \mid r \in R\}$ is an ideal of 'R'.
(ii) If I is an ideal of a ring 'R' with unity and $1 \in I$ then prove that $I=R$.

(b) Find all the principal ideals of the ring $R=\{0, 1, 2, 3, 4, 5\}$ w.r.t. $\oplus_6$ and $\otimes_6$ as two compositions.

OR

5. (a) If $f: R \to R'$ is a homomorphism of a ring R into R' then prove that
(i) $f(0)=0'$ where 0 and 0' are the zero elements of R and R' respectively.
(ii) $f(-a)=-f(a)\ \forall a \in R$.

(b) State and prove fundamental theorem of homomorphism of rings.

PART - C

Answer **two** full questions. **2x10=20**

6. (a) Find the constants a and b so that the surfaces $x^2+ayz=3x$ and $bx^2y+z^3=(b-8)y$ intersect orthogonally at the point $(1, 1, -2)$.

(b) If ϕ is a scalar point function and $\vec{f}$ is a vector point function then prove that $\operatorname{div}(\phi\vec{f}) = \phi(\operatorname{div}\vec{f}) + (\operatorname{grad}\phi)\cdot\vec{f}$

OR

7. (a) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then prove that $\nabla^2(r^3\vec{r})=18r\vec{r}$ where $r=|\vec{r}|$

(b) If $\vec{F}=\nabla\left(x^3+y^3+z^3-3xyz\right)$ then find $\nabla\cdot\vec{F}$ and $\nabla\times\vec{F}$

8. (a) Show that $\operatorname{div}(\vec{a}\times(\vec{r}\times\vec{a}))=2|\vec{a}|^2$ where $\vec{a}$ is a constant vector.

(b) Show that $\vec{F}=(x^2-yz)\hat{i}+(y^2-xz)\hat{j}+(z^2-xy)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F}=\nabla\phi$

OR

9. (a) (i) If $\vec{F} = 3xy\hat{i} + 20yz^2\hat{j} - 15xz\hat{k}$ and $\phi = xyz$, then find $\text{curl}(\phi\vec{F})$.

(ii) Show that $\vec{F} = 2x^2z\hat{i} - 10xyz\hat{j} + 3xz^2\hat{k}$ is solenoidal.

(b) Find $\text{curl}(\text{curl}\vec{F})$ if $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

PART - D

Answer **two** full questions. **2x10=20**

10. (a) Use the method of separation of symbols to prove that

$$u_0 - u_1 + u_2 - u_3 + \ldots.. + \infty = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \frac{1}{16}\Delta^3 u_0 + \ldots..$$

(b) Obtain a function whose first difference is $x^3 + 3x^2 + 5x + 12$.

OR

11. (a) Find the number of students from the following data who secured marks not more than 45.

Marks	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of Students	35	48	70	40	22

(b) Find a polynomial of lowest degree which assumes the values 10, 4, 40, 424, 620 at $x = -2$, 1, 3, 7 and 8 respectively, using Newton's divided difference formula.

12. (a) By employing Newton-Gregory backward difference formula, find $f(9.7)$ from the following data.

x	8	8.5	9	9.5	10
$f(x)$	50	57	64	71	75

(b) Using Simpson's $\frac{1}{3}^{rd}$ rule, Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ dividing the interval (0, 1) into 8 equal parts.

OR

13. (a) Applying Lagrange's formula find $f(5)$, given that $f(1)=2$, $f(2)=4$, $f(3)=8$ and $f(7)=128$.

(b) Using Simpson's $\frac{3}{8}^{th}$ rule, Evaluate $\int_4^{5.2} \log_e x\, dx$ taking $h=0.2$.

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